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13. ABSTRACT (Maximum 200 words)

This research effort has as its objectives: the experimental characterization and low order modeling of the coherent structure dynamics of high Reynolds number rectangular jets of low and moderate aspect ratios via a joint application of the proper orthogonal decomposition (POD) and wavelet analysis techniques. This includes the extraction and characterization of the u, v, and w-component POD eigenfunctions and associated eigenvalues. The rectangular jet dynamics (embodied in the POD temporal coefficients) is obtained by projection of the POD eigenfunctions onto instantaneous realizations of the flow field. This projection is performed via wavelet analysis. Similarly, Galerkin projection of a truncated subset of the empirical eigenmodes onto the Navier-Stokes equations is used to develop a low-order dynamical systems model of the rectangular jet flow field. The validity of assumptions associated with the treatment of the truncated modes and their effect on the coherent structure is established by comparing the dynamic behavior resulting from the model ODE's with those from projection of POD modes onto the actual flow. The dynamical system model of the rectangular jet will be utilized for the establishment of rectangular jet flow control strategies. The flow control algorithm will be demonstrated via both the model and laboratory jet flows.

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Dynamic Modeling and Flow Control Strategies for Rectangular Jets:
Application of POD and Wavelet Analysis

AFOSR Grant F49620-00-1-0013

Final Project Report

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Notre Dame, IN 46556

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Start Date: November 15, 1999

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UNIVERSITY of NOTRE DAME
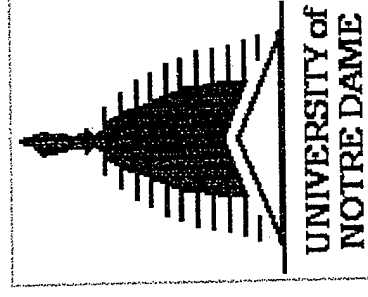
Organization

- Motivation and Objectives

- Approach

- Experimental Results

- Modeling Results

- Plans for the Upcoming Year

- Questions / Discussion

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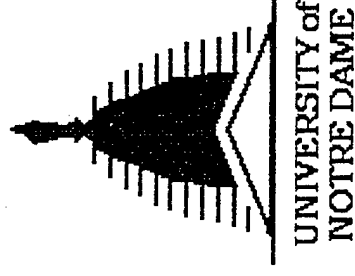
Motivation

- It is widely recognized that coherent vortical structures in turbulent flows are important in processes such as *scalar and momentum transport, mixing and noise generation.*

- The routine use of the coherent structure concept in turbulence modeling and flow control strategies has not yet been realized.

- Several recent successes at addressing coherent structure topology and dynamics have been achieved by the joint application of the proper orthogonal decomposition (POD) and dynamical systems theory.

- Example "success stories":

  Near–wall region of turbulent boundary layer,
  Aubry, N., Holmes, P., Lumley, J. L. and Stone, E., 1988

  Asymptotic region of plane mixing layer
  Ukeilley L. and Glauser, M. N., 1995

Objectives

- The experimental characterization and low-order modeling of the coherent structure dynamics of rectangular jets of low and moderate aspect ratios via a joint application of POD and wavelet analysis. (Rectangular jets are of interest from the standpoint of both thrust vectoring and low observables.)

- The resulting dynamical system model of the rectangular jet will be utilized for the establishment of rectangular jet flow control strategies.

Approach

- Perform initial year-one proof-of-concept experiments and modeling in the *far field of a planar jet*. These are considered prerequisite to the treatment of the more complex rectangular jet.

AR = 7

Two inhomogenous spatial
coordinates (y, z)

AR = 36
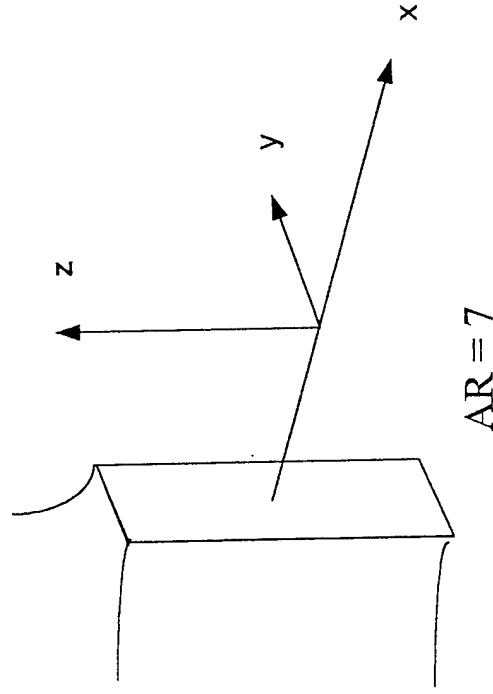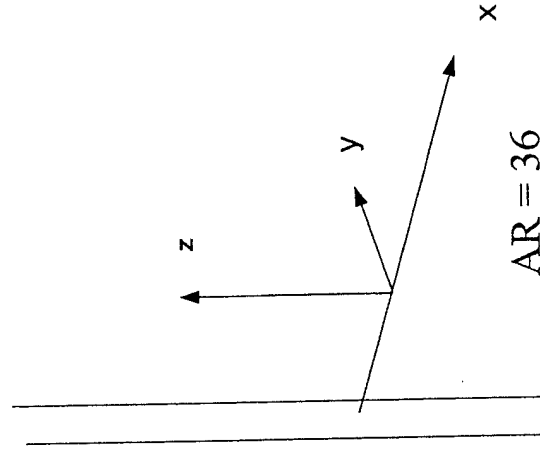
Single inhomogeneous
spatial coordinate, y

Approach

- The measurements were performed in the fully developed part of the flow ($50 \leq x/D \leq 90$) where visualization by injection of a tracer reveals no evidence of underlying coherent structure.

- *This made an ideal test case to investigate the potential of the method for extraction and modeling of flow structure in a complex environment.*

- Approach to be taken for the investigation of the rectangular jet is fundamentally similar to that which will be described for the proof-of-concept planar jet study.

Approach:

experiment

modeling

Apply POD to extract a set of spatial eigenmodes and associated eigenvalues from the rectangular jet.

Need to Recover Phase Information

Use the Galerkin method to project a truncated set of POD modes onto the Navier-Stokes equations . Use Heisenberg model for dissipative influence of unresolved modes.

Project POD Modes onto instantaneous epxerimental realizations of the flow

Model flow coherent structure dynamics

tune dissipative parameter

Experimentally obtained flow coherent structure dynamics

Compare

Use model & dynamical systems theory to investigate the dynamics of the candidate flow field.

Flow control

Planar Jet Flow facility

Nozzle width D=0.5"
Nozzle exit aspect ratio 36:1
Laminar nascent free shear layers
Initial velocity = 35 m/sec
Re=28,000

Acquisition System

Confining Plates

Screens and Honeycombs

Plenum Chamber

Jet Exit

Centrifugal Blower

X

Y

Z

Top View

8 ft

Traverse System

5.25 ft

X-Wire Rakes

Jet

Nozzle

Screens and Honeycombs

10 ft

4 ft

11"

Centrifugal Blower
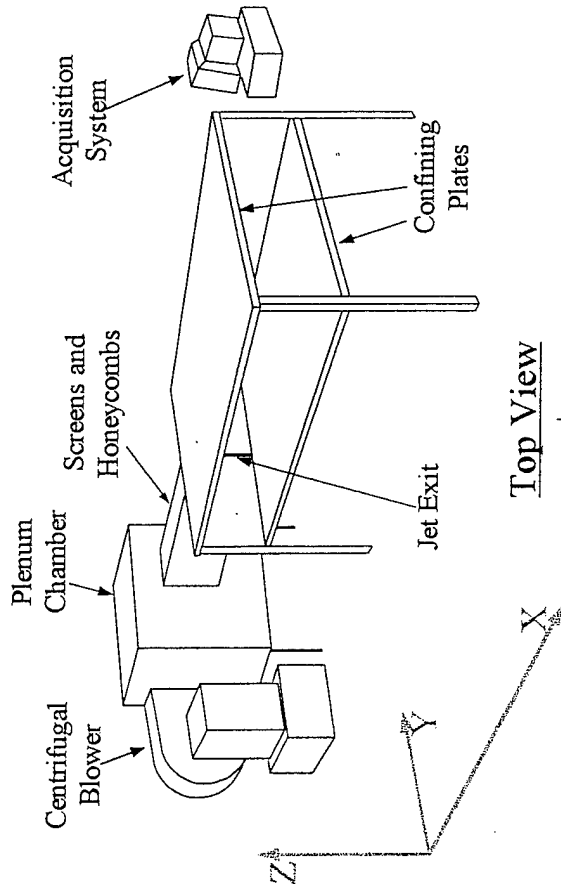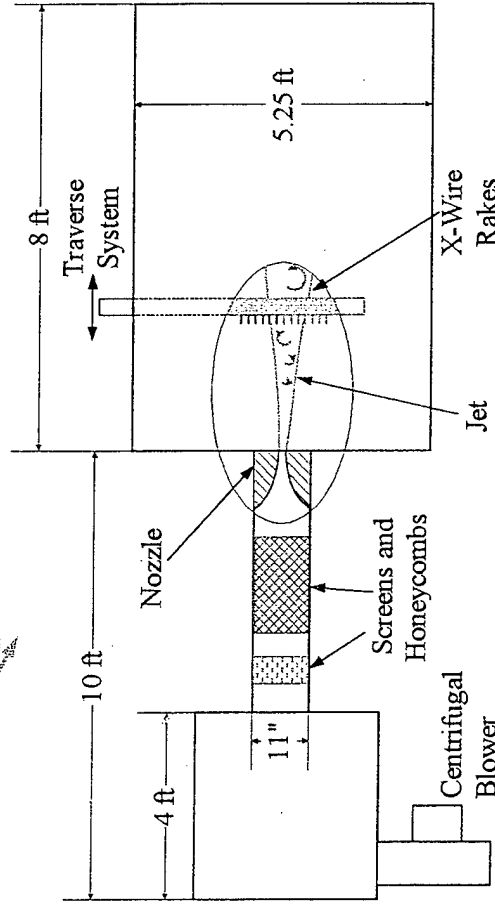
Flow exhibits self-similarity
in second-order moments
after x/D=50

Twin X-wire Rake Set-up

Rakes of X-wire probes

Data Acquisition System

z/D

x/D

Flow
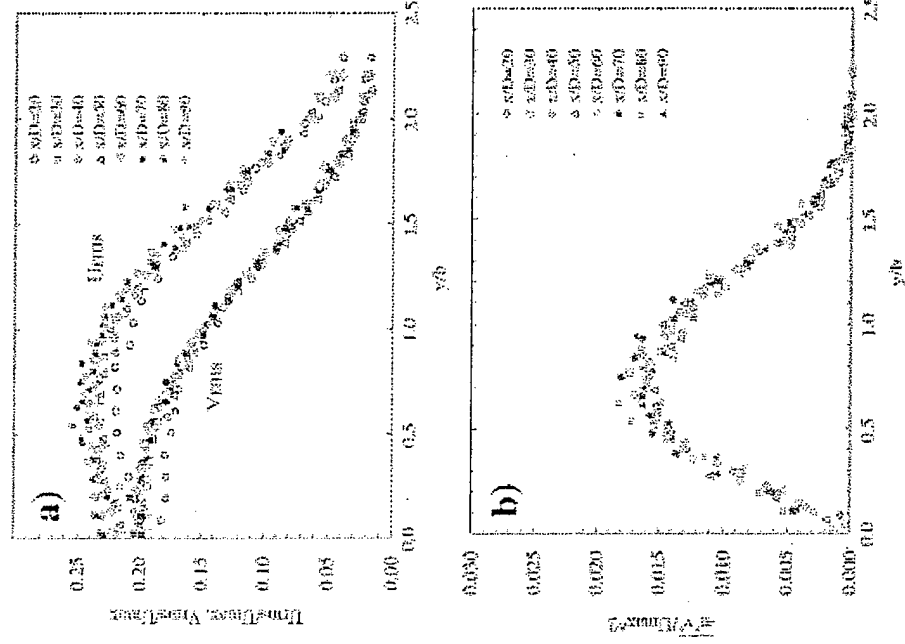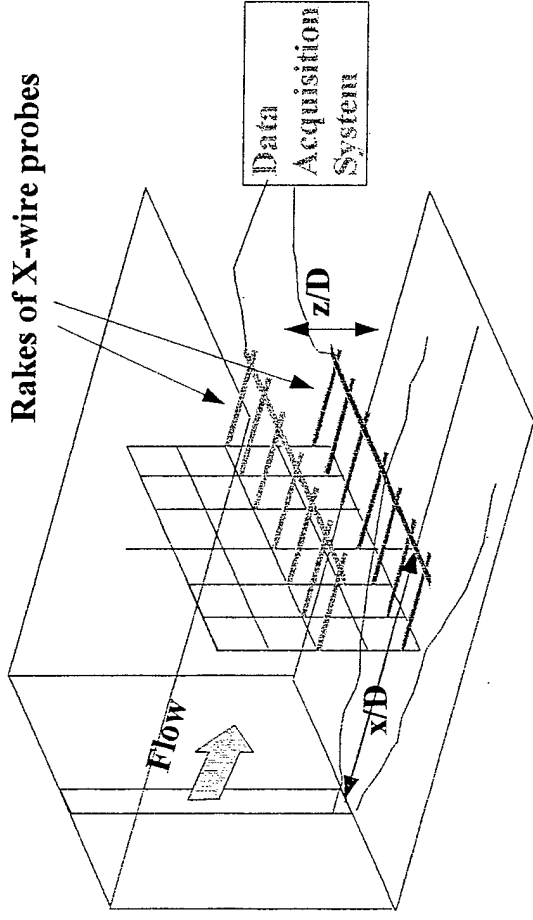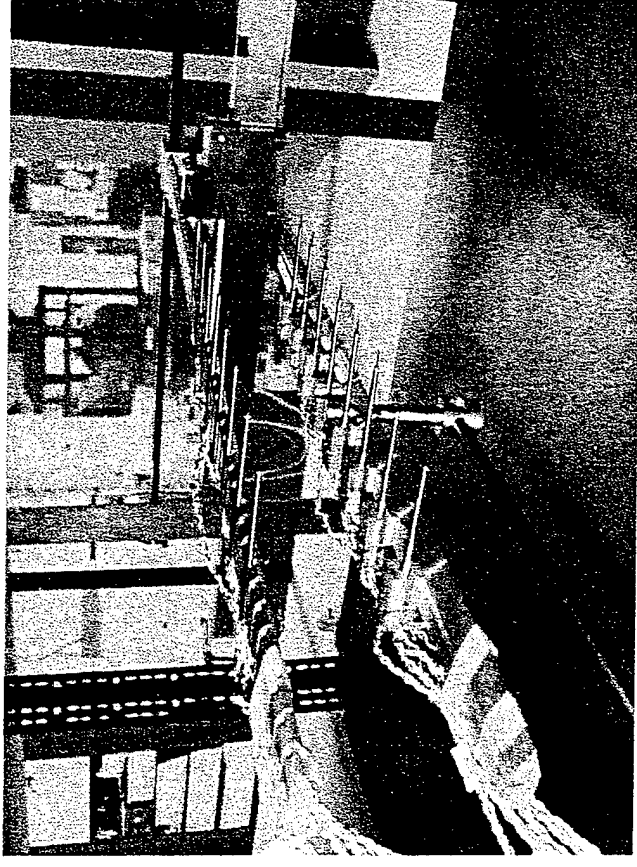
- 16 x-wire probes (32 sensors) mounted on two movable rakes.
- Spacing between probes - 2" (0.66b)
- Spacing between rakes - 15 positions in 0.75" (0.25b) increments.
- Blockage effect is demonstrated to be negligible.

Eigenmode Extraction

Rakes of X-wire probes

Data Acquisition System

Flow

z/D

x/D
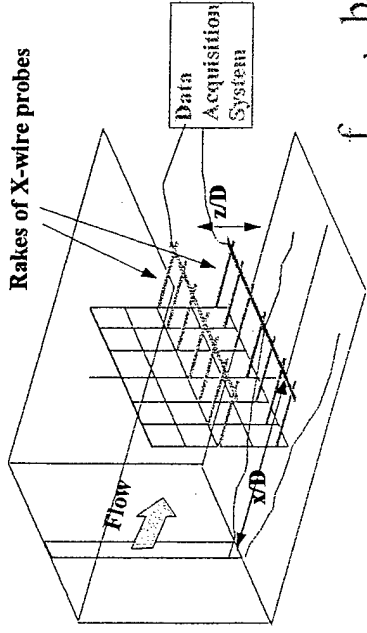
Instantaneous velocity components $u_\alpha(y,z,t)$

FT in time (500 blocks of 1024 points each)

$$\hat{u}_\alpha(y,z,f) = \int u_\alpha(y,z,t)e^{-2\pi i f t}\, dt$$

$f_{sample}\, b/U_m = 8.0$

Spectral Correlation Matrix $\quad S_{\alpha\beta}(y,y',\Delta z,f) = \left\langle \hat{u}_\alpha^*(y,z,f)\hat{u}_\beta(y',z+\Delta z,f)\right\rangle$

FT in z-direction (16 points)

$$\Phi_{\alpha\beta}(y,y';f,k_z) = \int S_{\alpha\beta}(y,y',\Delta z,f)e^{-2\pi i k_z \Delta z}\, d(\Delta z)$$

$$\int \Phi_{\alpha\beta}(y,y';f,k_z)\varphi_\beta^{(n)}(y';f,k_z)\,dy' = \lambda^{(n)}(f,k_z)\varphi_\alpha^{(n)}(y;f,k_z)$$

POD equation to find eigenvalues (energy) and eigenfunctions (spatial distribution) of the modes

* Similar to the implementation of Ukeiley & Glauser (1995)

Example: First POD Mode (x/D=70)

U-mode   V-mode   W-mode

$$\left|\varphi_\alpha^{(1)}(y, St, k_z b = 0)\right|$$

$$\left|\varphi_\alpha^{(1)}\left(y, St, \frac{k_z b}{2\pi} = 0.33\right)\right|$$

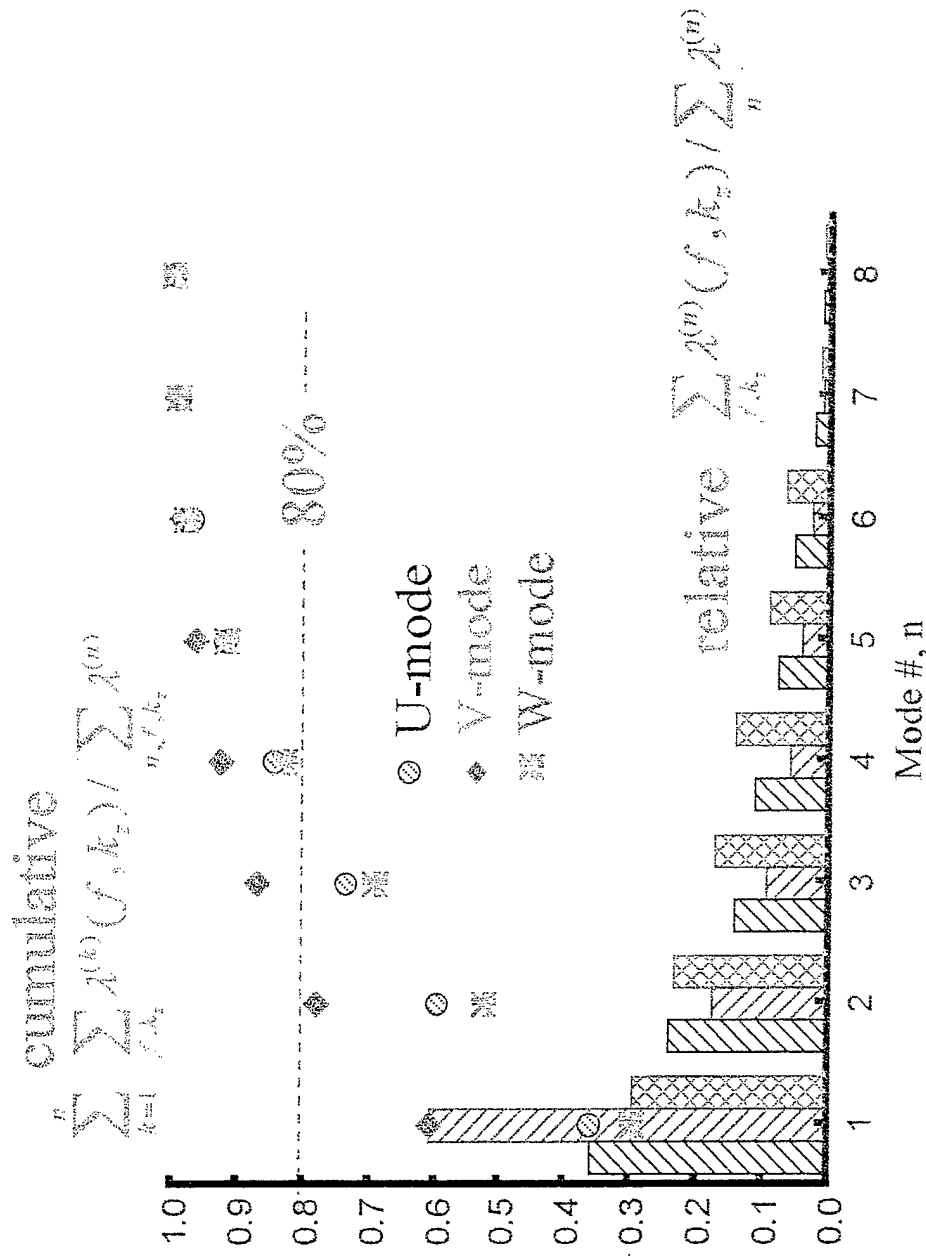$$\left|\varphi_\alpha^{(1)}\left(y, St, \frac{k_z b}{2\pi} = 0.5\right)\right|$$

Eigenvalue: $\lambda^{(1)}(St, k_z b)$
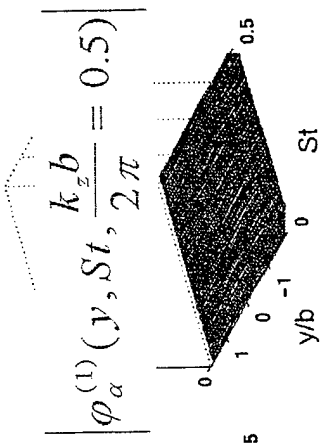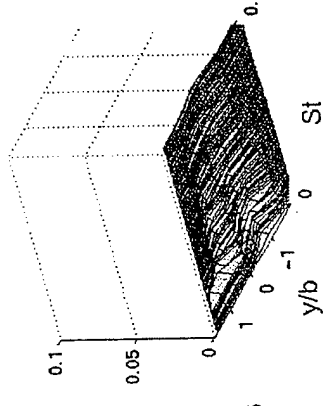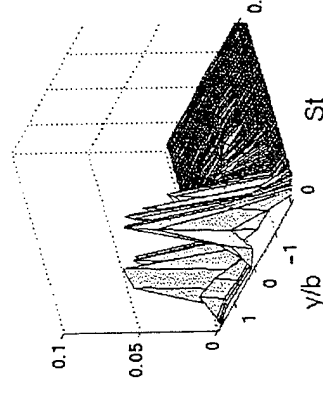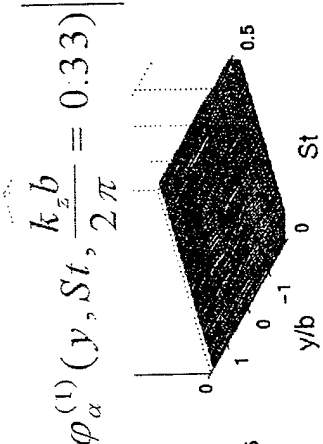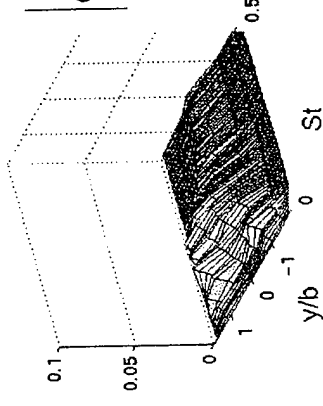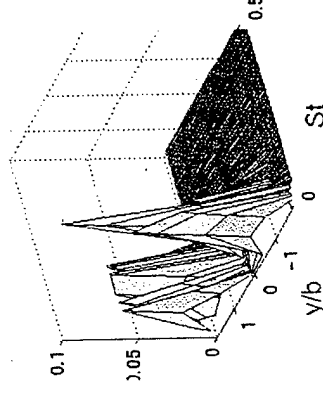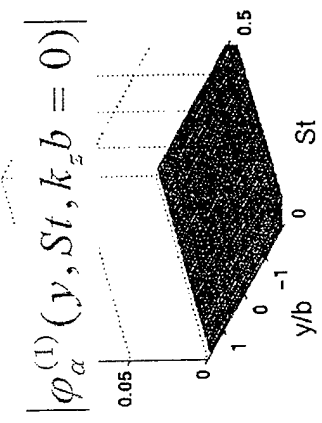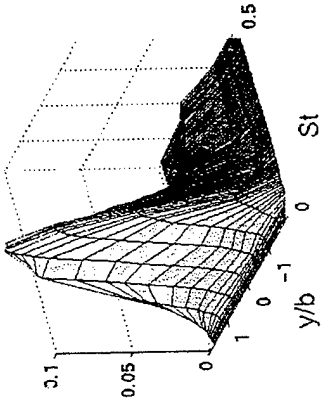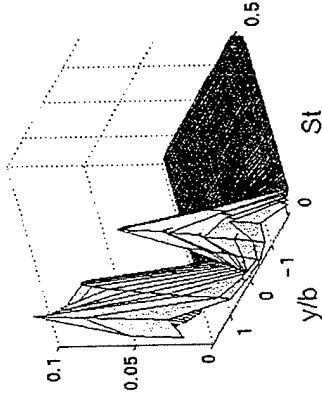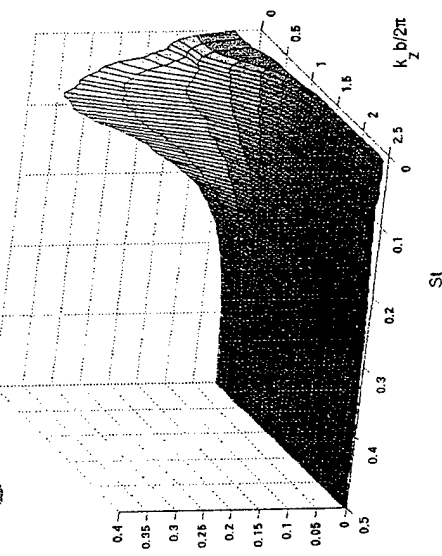
- Details presented in Gordeyev and Thomas, *JFM*, 414, pp. 145-194, 2000

Energy Convergence

$$\sum_{k_1}^{n} \sum_{k_3} \lambda^{(n)}(j,k_3) / \sum_{n} \lambda^{(n)}$$

cumulative

$$\sum_{k_1} \lambda^{(n)}(j,k_3) / \sum_{k_3} \lambda^{(n)}$$

relative

80%

- ⊙ U-mode
- ◆ V-mode
- ※ W-mode

Mode #, n

- Series converges rapidly; for example:
- 4 U-modes, 2 V-modes and 4 W-modes to capture over 80% of energy of their respective component.

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Velocity Field Reconstruction via Wavelets

$$\int \Phi_{\alpha\beta}(y,y';f,k_z)\varphi_\beta^{(n)}(y';f,k_z)dy' = \lambda^{(n)}(f,k_z)\varphi_\alpha^{(n)}(y;f,k_z)$$

• Eigenmodes are known up to an arbitrary phase $\exp(i\sigma(f,k_z))$

Need to have instantaneous flow field realizations

$$\tilde{u}_\alpha(y;k_z,\kappa(f),\tau) = \sum_n \underbrace{c^{(n)}(\kappa(f),\tau;k_z)}_{\text{unknown phases}}\varphi_\alpha^{(n)}(y;f,k_z) \quad \text{- Forward Wavelet}$$
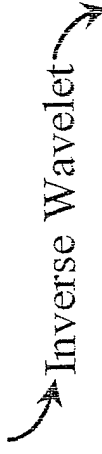
From orthogonality of the modes

$$c^{(n)}(k_z,\kappa(f),\tau) = \int \tilde{u}_\alpha(y;k_z,\kappa(f),\tau)\varphi_\alpha^{(n)*}(y;f,k_z)dy$$

Reconstruction of the individual mode

$$\tilde{u}_\alpha^{(n)}(y;k_z,\kappa(f),\tau) = c^{(n)}(\kappa(f),\tau,k_z)\varphi_\alpha^{(n)}(y;f,k_z)$$

Inverse Wavelet $\longrightarrow u_\alpha^{(n)}(y,k_z,t)$

Structures in physical space $u_\alpha^{(n)}(y,k_z,t)$

$$\boxed{u_\alpha(y,k_z,t) = \sum_n u_\alpha^{(n)}(y,k_z,t)}$$

$$\boxed{\begin{array}{l}\text{Need to perform FT in z-direction}\\ \text{Measurements with 3 rakes minimum}\\ \text{With 2 rakes - } k_z=0 \text{ only}\end{array}}$$

Velocity Reconstruction (planar mode)

$x/D=70$

$$\overline{U}_1(y) + \sum_n u_\alpha^{(n)}(y, k_z b = 0, -U_c t)$$
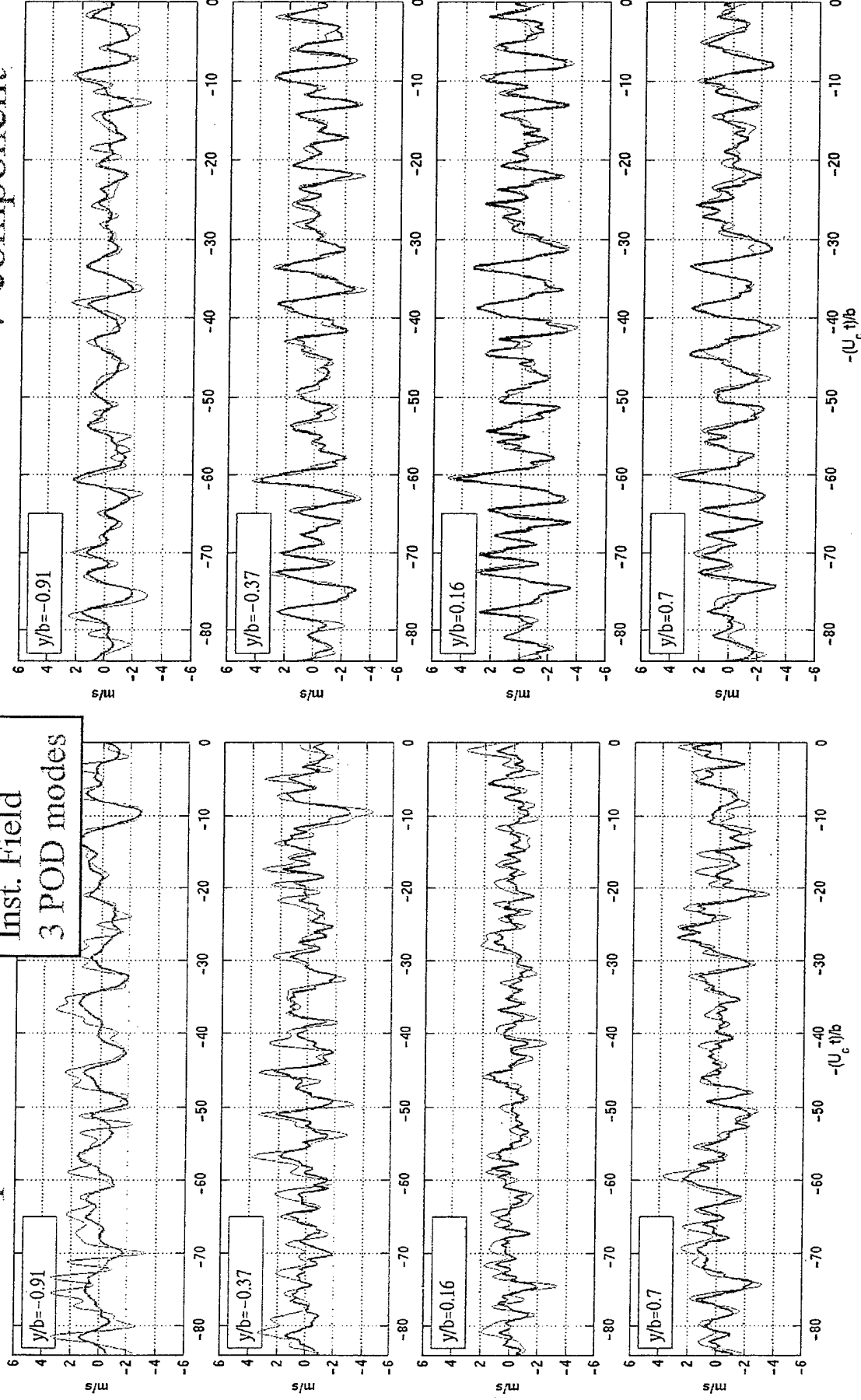
First 3 planar POD modes
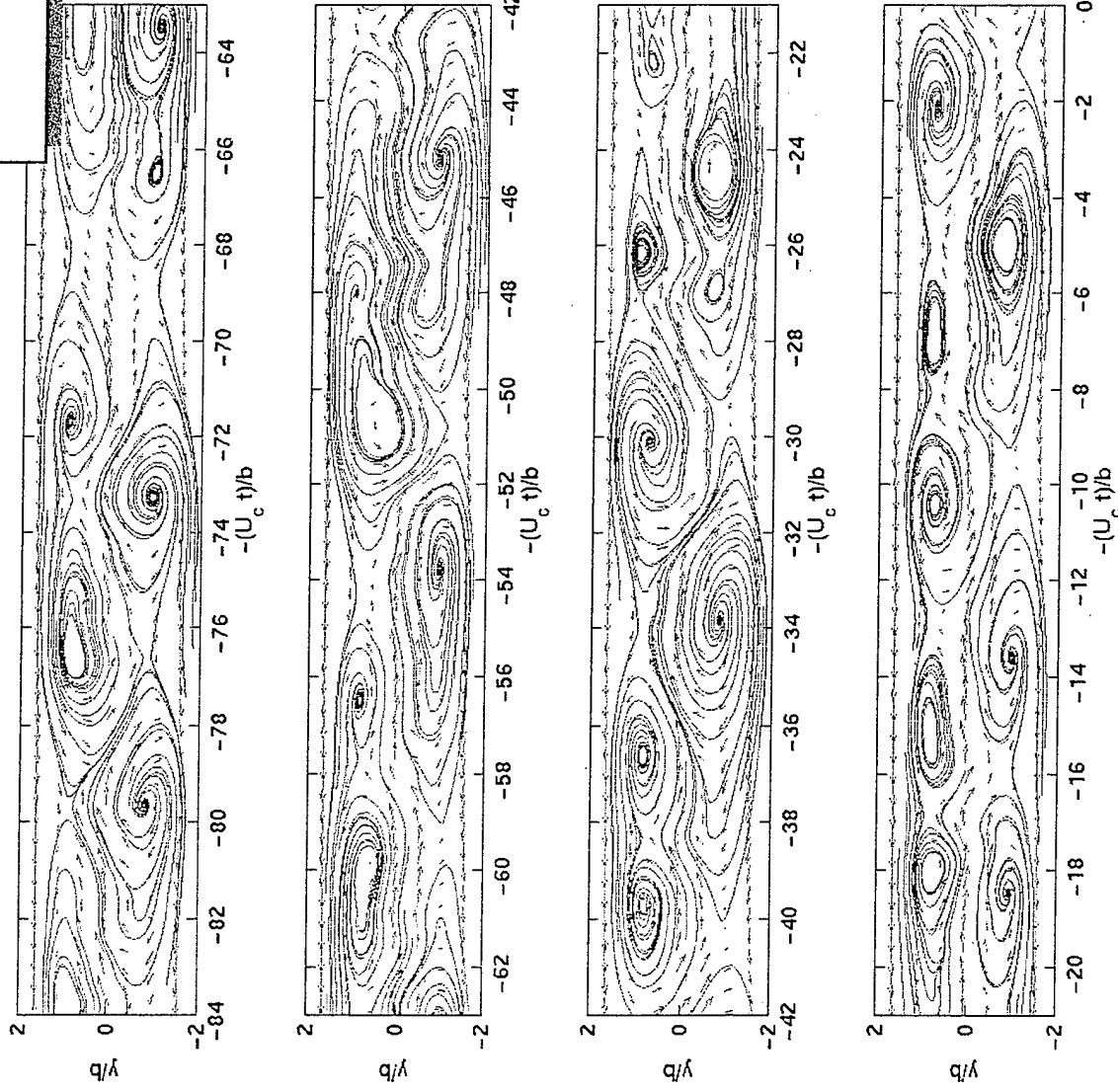
- Strong evidence of spanwise vortical structures arranged asymmetrically with respect to the jet axis.

- Karman street like organization.

- Weakly intermittent appearance.
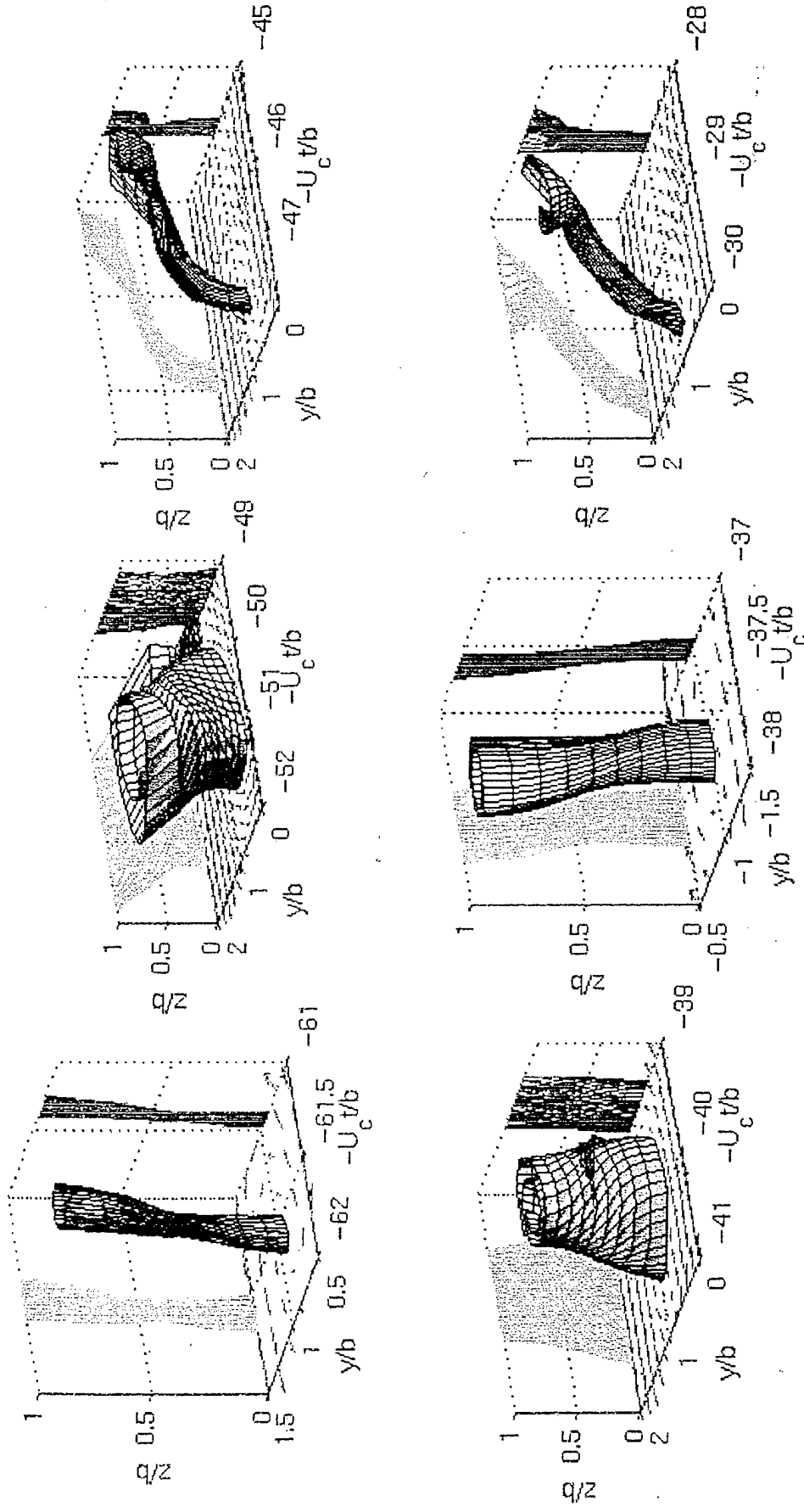
Sample Velocity Traces

u-component

v-component

- First 3 POD modes capture velocity signals quite well.

Reconstruction of non-planar structure

- Effect of non-planar modes is to tilt and bend the spanwise vortices in the crosstream and streamwise direction
- Non-planar modes appear to present a small disturbance to a dominant planar structure

Galerkin Projection and Truncation

Governing Equations: $L(u) = \dfrac{\partial u}{\partial t} + L(u) + Q(u,u) = 0$

Truncation: $u^N(\vec{x},t) = \displaystyle\sum_{n=1}^{N} a_n(t)\varphi_n(\vec{x})$ ← from experiment

Require, $(L(u^N) \cdot \varphi_n) = 0$

Finally, $\dfrac{da_n}{dt} = Ba_n + Q(a_n, a_n) + C(a_n, a_n, a_n)$

production	interaction	interaction
dissipation	between	between modes and
convection	modes	mean flow

• Need to take into account energy transfer between resolved modes and unresolved turbulence (simple eddy viscosity model)

dissipation rate $\sim \nu + \alpha\nu_T$

Aubry, Holmes, Lumley, Stone, 1988 - boundary layer
Zheng, Glauser, 1991 - round jet
Ukeiley, Glauser, 1995 - planar shear layer

Modeling Approach

$$\varphi_{\alpha}^{(n)} = \varphi_{\alpha}^{(n)}(y, k_x, k_z)$$ - obtained from the experiment

<u>Key Assumptions:</u>

- Homogeneous flow in x-direction, $\quad \overline{U}_1 = \overline{U}_1(y)$

$$t \rightarrow x = -U_c t, \quad \varphi(y, f, k_z) \rightarrow \varphi(y, -k_x, k_z)$$

- $U_c = 0.7 U_M = \text{const}$

- $U_2 = U_3 = 0$

- $n = 1$ - first POD mode only

- no feedback between mean flow and resolved modes

Symmetry Conditions

-translation in x-direction

$$\varphi_{u,v}^{(n)}\left(v, k_x, k_z\right) = \varphi_{u,v}^{(n)*}\left(v, -k_x, k_z\right)$$

$$\varphi_{w}^{(n)}\left(v, k_x, k_z\right) = -\varphi_{w}^{(n)*}\left(v, -k_x, k_z\right)$$

$$a^{(n)}\left(t, v, k_x, k_z\right) = a^{(n)*}\left(t, v, -k_x, k_z\right)$$

-reflection in z-direction

$$\varphi_{u,v}^{(n)}\left(v, k_x, k_z\right) = \varphi_{u,v}^{(n)}\left(v, k_x, -k_z\right)$$

$$\varphi_{w}^{(n)}\left(v, k_x, k_z\right) = -\varphi_{w}^{(n)}\left(v, k_x, -k_z\right)$$

$$a^{(n)}\left(t, v, k_x, k_z\right) = a^{(n)}\left(t, v, k_x, -k_z\right)$$

k_z

k_x

resolved modes

from symmetry

streamwise modes

spanwise modes
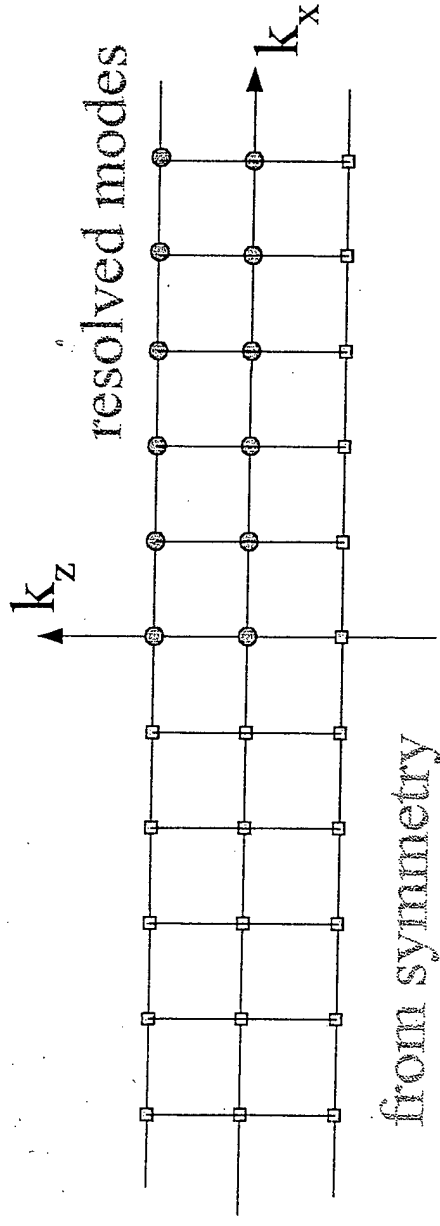
# of equations=2 x 1 x 50

Model behavior versus α
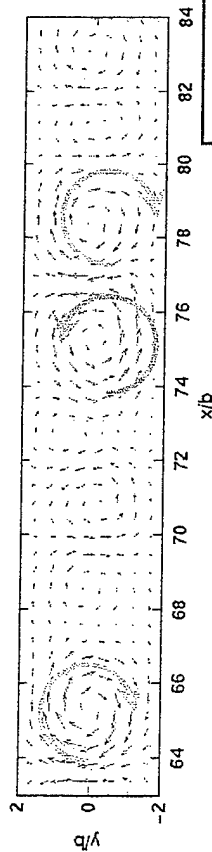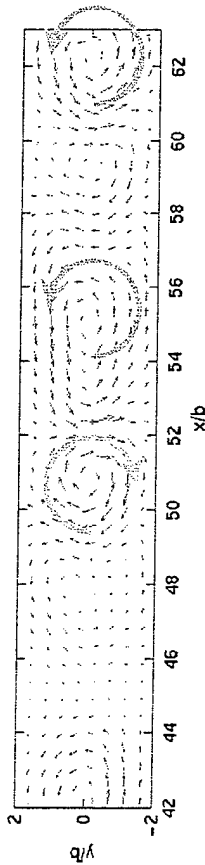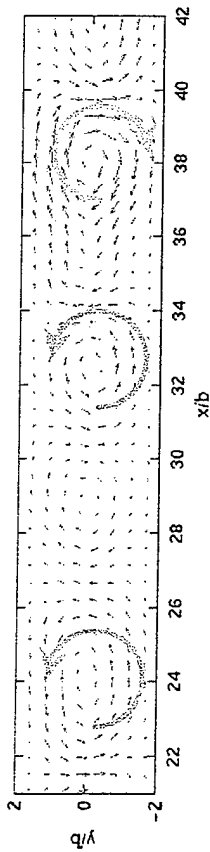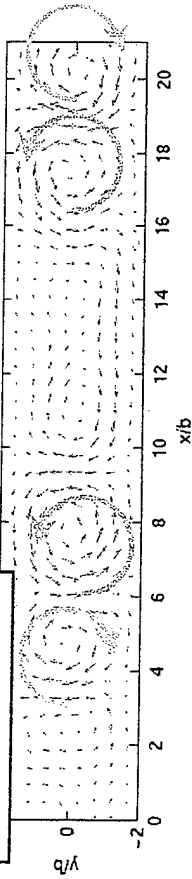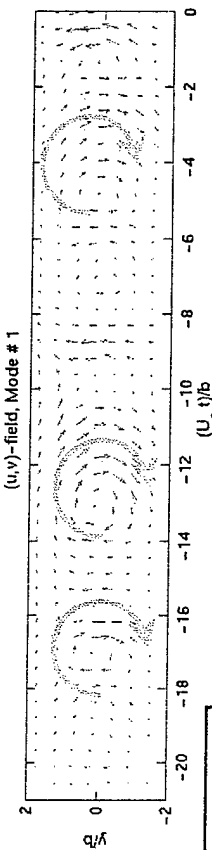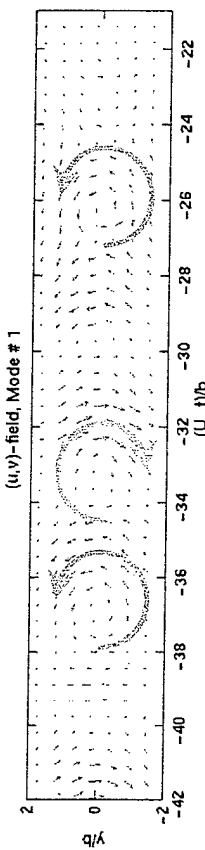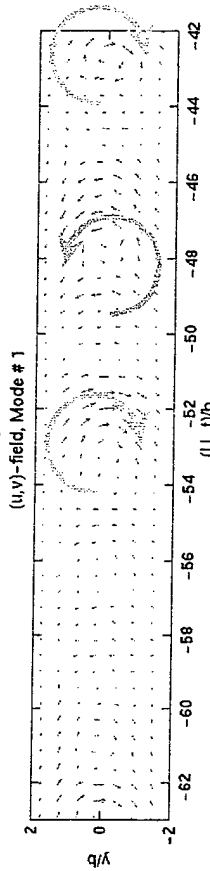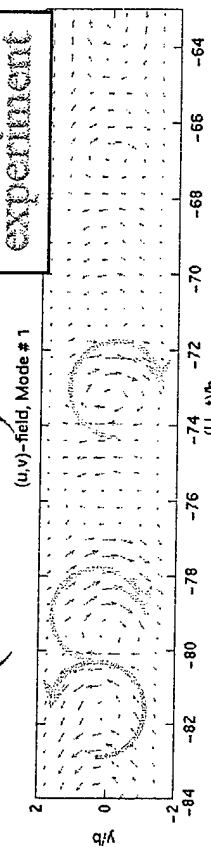
infinite   pseudo-periodic   periodic   trivial

$\alpha=10$ $\alpha=32$ $\alpha=40$

Reconstructed flow (model)

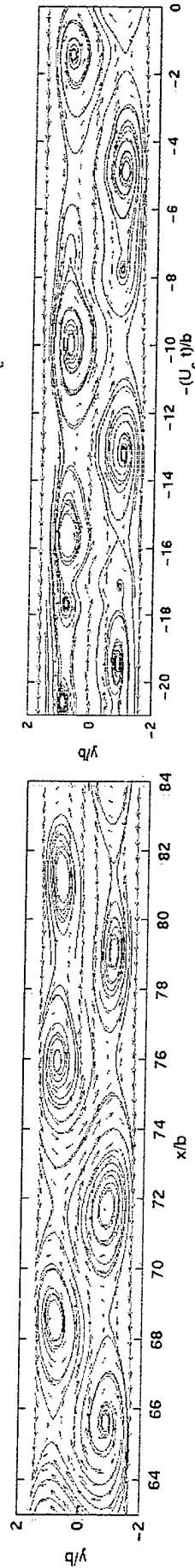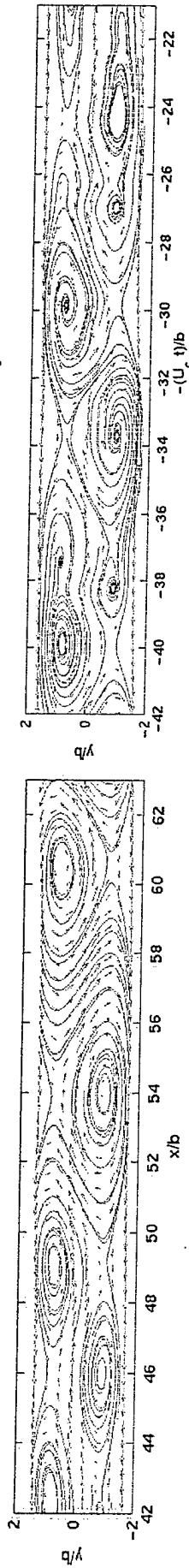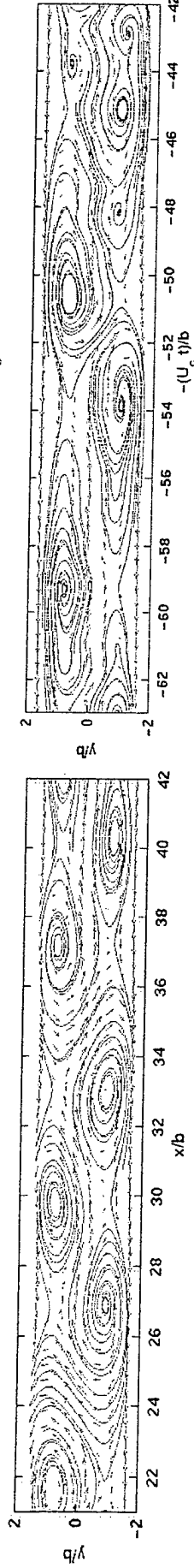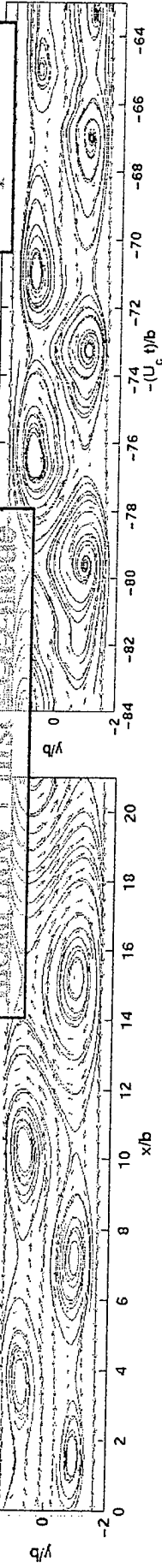model, α=15

experiment

first POD mode

- Pseudo-periodic regime agrees very well with experimentally observed topology of large-scale coherent structures in the jet.
- Model correctly predicts lateral sweeping motions, as well as the typical size, spacing and locations of vortical structures
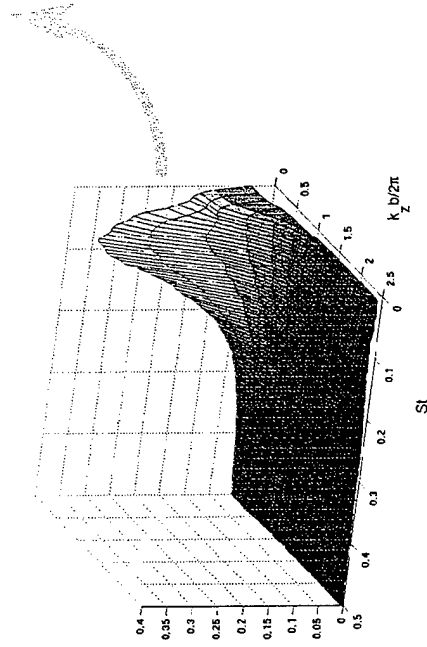
AFOSR Contractors' Meeting on Turbulence, 8/16/2000

Reconstructed flow (model)

- Pseudo-periodic regime agrees very well with experimentally observed topology of large-scale coherent structures in the jet

- Model correctly predicts lateral sweeping motions, as well as the typical size, spacing and locations of vortical structures
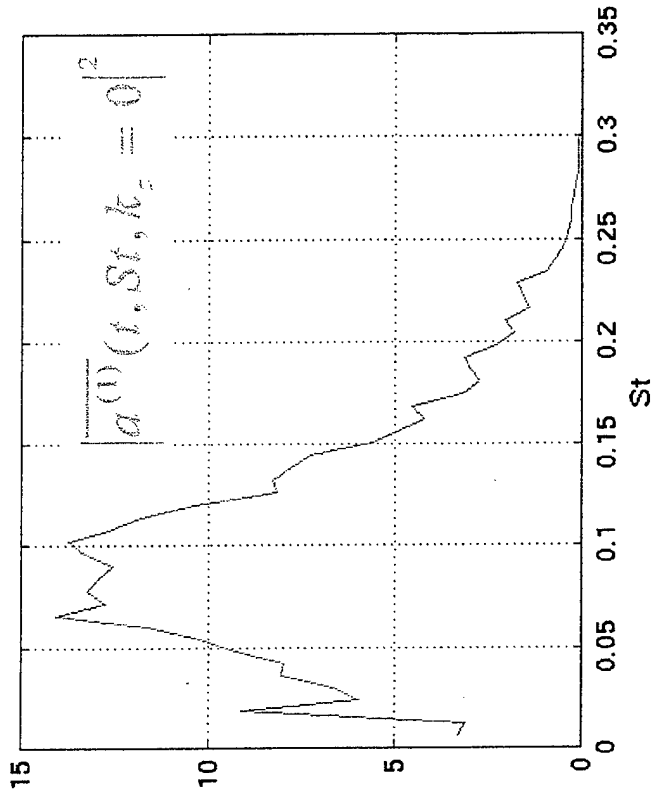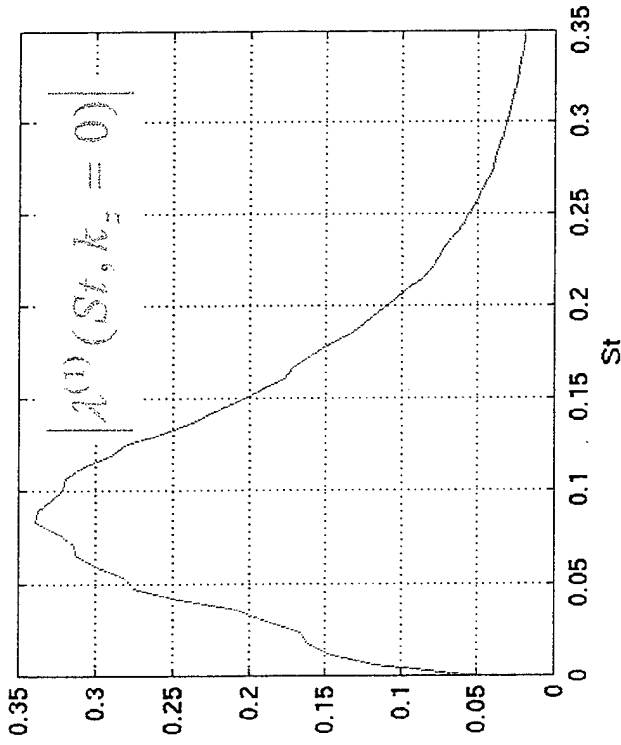
- Agreement between model and experiment - weak interaction between mean flow and structures

Spectrum Recovery

$$\left| A^{(1)}(St, k_z = 0) \right|$$

$$\overline{\left| a^{(1)}(t, St, k_z = 0) \right|^2}$$

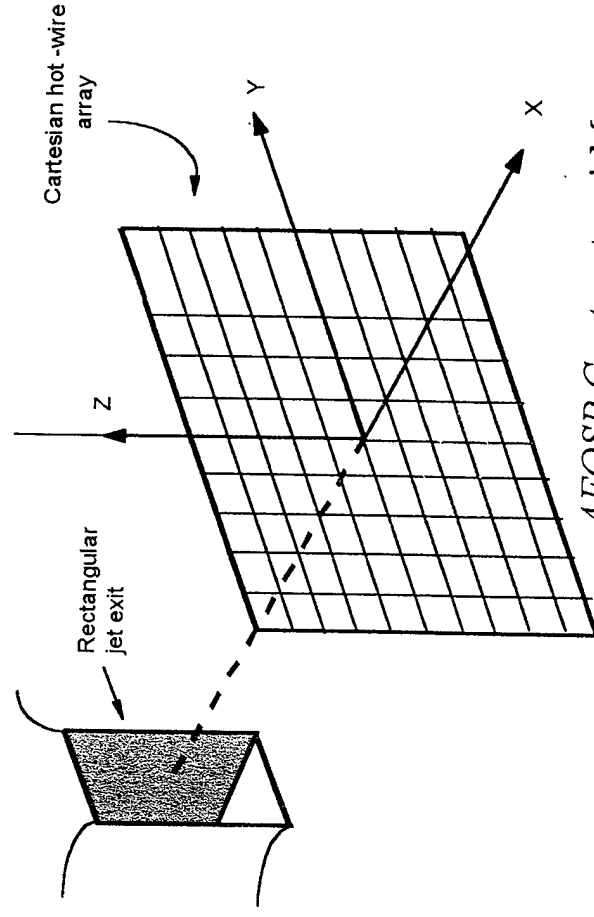- Model correctly recovers the shape of the spectrum

Project Status

- Proof-of-concept experimental implementation of POD and low-order modeling of the self-similar planar turbulent jet has nearly been completed.

- Results show great promise for incorporating coherent structure dynamics in models of complex turbulent flows (like the rectangular jet of interest here).

- Presently working on the incorporation of nonplanar modes (first results obtained last week).

- Similar methodology to be applied to the AR=7 rectangular jet next.

Project Status

- Rectangular jet nozzle fabricated.

- Initial hot-wire surveys at Re = 50,000 now underway.

- Measurements to obtain the POD eigenfunctions $\varphi_\alpha^{(n)}(y, z, St)$ will commence soon.

Rectangular jet exit

Cartesian hot-wire array

Z

Y

X